Edge tunneling in fractional quantum Hall regime

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We address the issue of an apparent disagreement between theory and experiment on the I-V characteristic of electron tunneling from a metal to the edge of a two-dimensional electron gas in the fractional quantum Hall regime. A part of our result is a theory for the edge of the half-filled Landau level.

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Recently Grayson *et al* [1] systematically studied the current-voltage (I-V) characteristic for electron tunneling between a metal and the edge of a two dimensional electron gas (2DEG) at fractional filling factors (ν).

They found that for $1/4 \leq \nu \leq 1$, $I \sim T^{\alpha-1}V$ for $k_BT >> eV$, and $I \sim V^\alpha$ for $eV >> k_BT$, where $\alpha \approx 1/\nu$. For primary filling factors such as $\nu = 1/3$, this result agrees with the theoretical prediction based on the assumption of sharp edge [2]. However for other filling factors such as $\nu = \frac{n}{2n+1}$, the experimental value ($\alpha \approx 1/\nu$) *disagrees* with the sharp-edge prediction: $\alpha = 3$. [2] Perhaps what is even more surprising is that scaling behavior in I-V was found even at filling fractions between Hall plateaus, and at $\nu = 1/2$. [1,3] Several attempts has been made to explain this result [4–6].

The purpose of the present paper is to address the above disparity between theory and experiment. However instead of dealing with an arbitrary filling factor satisfying $\nu < 1$, we shall limit ourselves to those correspond to incompressible fractional quantum Hall states and $\nu = 1/2$. Our working assumption is that the edge modes can be described by a quadratic action and for the most part we shall ignore dissipation.

The edge at incompressible filling factors: Two facts at the edges of real samples may contribute to the disagreement between experiment and theory.

1) The edge potentials in real samples are smooth. Moreover due to Coulomb interaction the edge can "reconstruct". When that happens there are pairs of left and right moving edge branches, and the exponent α becomes non-universal because it depends on the interaction between these counter-propagating modes. [7] Although edge reconstruction can explain the departure of α from the sharp-edge prediction, it cannot explain why $\alpha \approx 1/\nu$.

2) In reality charge and neutral modes have very different velocities. In particular, for reconstructed edge the neutral-mode velocities v_n are very small due to the flatness of the effective potential. On the contrary the charge velocity v_c is of order $e^2/\hbar$ due the long range Coulomb interaction. In the following we shall demonstrate that when temperature (T) and voltage (V) fulfill the condition that $v_c\Lambda_c >> k_BT, eV >> v_n\Lambda_n$ (here $\Lambda_{c,n}$ are the momentum cutoffs of the charge and neutral modes), the neutral modes do not contribute to the I-V exponent α. However, they are crucial in restoring the proper short time behavior of the electron Greens function ($G_e(x,t)$) required by the electron Fermi statistics. Due to the smallness of v_n, it is possible that Grayson *et al* are probing the edge dynamics above the energy scale $v_n\Lambda_n$. In that case only charge mode contributes to the tunneling exponent, and hence $\alpha = 1/\nu$.

The momentum cutoffs $\Lambda_{c,n}$ discussed above may have several different origins. For example it can arise from 1) the fuzziness in the position of tunneling events, and 2) the fact that above certain momentum the edge modes become over-damped. (Toward the end of this paper we will demonstrate that over-damping removes edge modes from contributing to the long-time decay of $G_e(x,t)$.) While the first mechanism gives the same cutoff for charge and neutral modes, the second can give $\Lambda_n << \Lambda_c$. This is because for the charge mode there is an extra conservation law - the charge conservation - that limits the number of decaying channels. Indeed as was demonstrated experimentally, the charge mode remains well defined at relatively high energies.

It has been pointed out in Ref. [5,6] that $\alpha = 1/\nu$ can be obtained if one assumes that the tunneling charges are added to the charge mode only. A difficulty with this explanation is that for ν^{-1} not equal to an odd integer this predicts an electron Greens function that does not respect the electron Fermi statistics. The main purpose of the the following section is to demonstrate a mechanism by which only the charge mode contributes to the long time decay of $G_e(x,t)$ (and hence α), but both charge and neutral modes contribute to ensure the correct short-time fermionic behavior of $G_e(x,t)$.

The two-mode model:

As far as electron tunneling is concerned it is sufficient to focus on only two, a neutral and a charge, edge modes. In the limit that the charge velocity (v_c) is much greater than all the neutral ones (v_n) the action assumes a very

simple form:

$$S = S_c + S_n$$
$$S_c = \frac{1}{4\pi\nu}\int \frac{d\omega dk}{(2\pi)^2} k(-i\omega + v_c k)|\phi_{k\omega}|^2$$
$$S_n = \frac{1}{4\pi\eta}\int \frac{d\omega dk}{(2\pi)^2} k(-i\omega + v_n k)|\chi_{k\omega}|^2. \quad (1)$$

Here ϕ and χ are the chiral fields associated with the charge and neutral modes respectively. (For purpose of regularization we shall keep v_n and take the limit $v_n \to 0$ at the end.) The electron operator is given by

$$\psi \sim e^{-\frac{i}{\nu}\phi} e^{-\frac{i}{\eta}\chi}. \quad (2)$$

The electron Fermi statistics requires

$$e^{i\pi(\frac{1}{\nu}+\frac{1}{\eta})} = -1. \quad (3)$$

The following is a sketch of how to obtain Eq.(1) from the multi-mode action for the edge of a fractional FQH state. [2] First, we identify the linear combination of the chiral fields corresponding to the total charge displacement, and call it ϕ. Second, we identify a different linear combination χ so that the electron annihilation operator can be written as Eq.(2). Third, we integrate out all the other linearly-independent chiral fields. In general the result is rather complicated. In the limit that the energy associated with neutral mode displacement is much smaller than the that of the charge mode we obtain Eq.(1).

Since the action in Eq.(1) is quadratic, the electron Greens function is given by

$$G_e(x,t) = e^{[G_\phi(x,t)-G_\phi(0,0)]/\nu^2} e^{[G_\chi(x,t)-G_\chi(0,0)]/\eta^2}, \quad (4)$$

where

$$G_\phi(x,t) = \int \frac{dk d\omega}{(2\pi)^2} e^{i(kx-\omega t)} \frac{2\pi\nu}{k(-i\omega + v_c k)}$$
$$G_\chi(x,t) = \int \frac{dk d\omega}{(2\pi)^2} e^{i(kx-\omega t)} \frac{2\pi\eta}{k(-i\omega + v_n k)}. \quad (5)$$

After performing the frequency integral and set $v_n = 0$ we obtain

$$G_\phi(x,t) = 2\pi\nu \int_0^\infty \frac{dk}{(2\pi)} \{ \frac{\cos kz}{k} + i\mathrm{sign}(t) \frac{\sin kz}{k} \}$$
$$G_\chi(x,t) = 2\pi\eta \int_0^\infty \frac{dk}{(2\pi)} \{ \frac{\cos kx}{k} + i\mathrm{sign}(t) \frac{\sin kx}{k} \}. \quad (6)$$

In the above $z \equiv x + i v_c t$. First we note that

$$G_\phi(x,0^+) - G_\phi(x,0^-) = i\pi\nu \mathrm{sign}(x)$$
$$G_\chi(x,0^+) - G_\chi(x,0^-) = i\pi\eta \mathrm{sign}(x). \quad (7)$$

It is Eq.(7) that ensures the correct short time behavior of $G_e(x,t)$. By putting $x = 0^+$ it is simple to show that

$$G_\phi(0^+,t) \sim e^{i\eta\frac{\pi}{2}\mathrm{sign}(t)} e^{[G_\phi(x,t)-G_\phi(0,0)]/\nu^2}$$
$$\sim e^{i\eta\frac{\pi}{2}\mathrm{sign}(t)} (\frac{1}{t})^{1/\nu}. \quad (8)$$

Eq.(8) has the property that while its short time behavior complies with the Fermi statistics, its scaling exponent, $1/\nu$, is solely determined by the charge mode.

In reality with small but non-zero neutral mode velocities, we expect Eqs (1) and (8) to apply as long as $(v_c\Lambda_c)^{-1} << |t| << (v_n\Lambda_n)^{-1}$, where $\Lambda_{c,n}$ stands for the momentum cutoffs of edge modes. In terms of the experimental parameters it says that when $v_c\Lambda_c >> k_B T, eV >> v_n\Lambda_n$ the I-V exponent α is $1/\nu$.

The edge at half-filling: Now we turn to $\nu = 1/2$ where gapless bulk excitations exist. In this case even the meaning of edge mode is unclear. In the following we spend some space to clarify this issue.

An electron has two basic attributes: its charge e and its Fermi statistics. In an earlier work on $\nu = 1/2$ one of us describes an electron as a boson carrying charge e and flux (statistical) ϕ_0. [8] In this picture the electron liquid is the superposition of a charge and a flux liquid. As an electron tunnels in both liquids expand. The two edge modes we shall encounter shortly are the edge deformation associated with these two liquids. We note that in the bulk this description reduces to the recent dipole theory of Refs. [9,8,10,11].

In the limit of vanishing electron bare effective mass, the low-energy action obtained by integrating out inter-Landau-level excitations is given by [8]

$$S = S_{int}[\frac{\nabla \times \boldsymbol{a}}{4\pi}] + i\int d^2x dt \{ \frac{1}{8\pi} \boldsymbol{a} \times \dot{\boldsymbol{a}} + \boldsymbol{a}\cdot\boldsymbol{j}$$
$$-\boldsymbol{b}\cdot\boldsymbol{j} - \frac{1}{4\pi}\boldsymbol{b}\times\dot{\boldsymbol{b}} \}. \quad (9)$$

In Eq. (9): $\boldsymbol{j} = \sum_i \dot{\boldsymbol{r}}_i \delta(\boldsymbol{x} - \boldsymbol{r}_i)$ where $\{\boldsymbol{r}_j(t)\}$ are the *electron* coordinates; $\hat{z}\times\boldsymbol{a}(\boldsymbol{x},t)$, $\hat{z}\times\boldsymbol{b}(\boldsymbol{x},t)$ are proportional to the charge and flux fluid displacements with $\frac{1}{4\pi}(\nabla\times\boldsymbol{a}, \hat{z}\times\dot{\boldsymbol{a}})$ and $\frac{1}{2\pi}(\nabla\times\boldsymbol{b}, \hat{z}\times\dot{\boldsymbol{b}})$ being the induced charge and flux 3-currents; the term S_{int} is given by

$$S_{int}[\delta\rho] = \int d^2x dt W(\boldsymbol{x})\delta\rho(\boldsymbol{x},t)$$
$$+\frac{1}{2}\int dt d^2x d^2x' V(\boldsymbol{x}-\boldsymbol{x}')\delta\rho(\boldsymbol{x},t)\delta\rho(\boldsymbol{x}',t), \quad (10)$$

where $W(\boldsymbol{x})$ is the confining potential and $V(\boldsymbol{x}-\boldsymbol{x}')$ is the electron-electron interaction. The partition function is given by $Z = \int' D[\boldsymbol{r}_j]D[\boldsymbol{a}]D[\boldsymbol{b}]exp\{-S\}$, where $\int'$ denotes the Feynman path integral over $\{\boldsymbol{r}_j\}$ and the functional integral over $\boldsymbol{a}$ and $\boldsymbol{b}$ under the constraint

$$\frac{1}{4\pi}\nabla\times\boldsymbol{a} = j_0 - \bar{\rho}, \quad \frac{1}{2\pi}\nabla\times\boldsymbol{b} = j_0, \quad (11)$$

where $j_0 = \sum_j \delta(\boldsymbol{x} - \boldsymbol{r}_j)$.

To separate the compressional and shear deformation of the charge and flux liquids we write

$$\boldsymbol{a} = \boldsymbol{a}_t + 2\nabla\phi$$
$$\boldsymbol{b} = \boldsymbol{b}_t + \nabla\chi, \tag{12}$$

where $\boldsymbol{a}_t$ and $\boldsymbol{b}_t$ are pure transverse vector fields. (By construction $\nabla\phi$ and $\nabla\chi$ do not affect the bulk charge and flux densities.) By substituting Eq.(12) into Eq.(9) we obtain

$$\begin{aligned}
S &= S_b + S_e + S_m + S_t\\
S_b &= S_{int}[\frac{\nabla\times\boldsymbol{a}_t}{4\pi}] + i\int_{y>0} d^2xdt\{\frac{1}{8\pi}\boldsymbol{a}_t\times\dot{\boldsymbol{a}}_t + \boldsymbol{a}_t\cdot\boldsymbol{j}\\
&\quad -\boldsymbol{b}_t\cdot\boldsymbol{j} - \frac{1}{4\pi}\boldsymbol{b}_t\times\dot{\boldsymbol{b}}_t\}\\
S_e &= \int dtdx\{\frac{1}{2\pi}[i\partial_t\phi\partial_x\phi + v_c(\partial_x\phi)^2] - \frac{i}{4\pi}\partial_t\chi\partial_x\chi\}\\
S_m &= i\int dtdx(\hat{y}\cdot\boldsymbol{j}_t)(2\phi-\chi)\\
S_t &= i\int d^2xdt(2\phi-\chi)(\partial_\mu j_\mu),
\end{aligned}\tag{13}$$

where $\boldsymbol{j}_t = \boldsymbol{j} - \frac{1}{4\pi}\hat{z}\times\dot{\boldsymbol{b}}_t = \boldsymbol{j} - \frac{1}{2\pi}\hat{z}\times\dot{\boldsymbol{a}}_t$ is the transverse component of the bulk current $\boldsymbol{j}$. In obtaining Eq. (13) we have adopted the geometry that the 2DEG occupies the space $y > 0$. We note that since only the charge displacement couples to the confining and Coulomb potentials, only the charge mode acquires a non-zero edge velocity. Again for regularization purposes we shall keep a neutral mode velocity v_n and set it to zero at the end.

In Eq.(13) S_m describes the mixing between the edge and bulk, and S_t describes the effect of tunneling on the edge displacements. (In the following due to the consideration of the electron Greens function we shall limit ourselves to a special tunneling event where an electron is added (removed) at time $t=0$ ($t=\tau$) at spatial position $\boldsymbol{x}=0$. In that case $\partial_\mu j_\mu = \delta(\boldsymbol{x})[\delta(t)-\delta(t-\tau)]$.) By setting $\delta S/\delta\phi = \delta S/\delta\chi = 0$ we obtain

$$\begin{aligned}
&\frac{1}{2\pi}(i\partial_t\partial_x\phi + v_c\partial_x\partial_x\phi) = \delta(\boldsymbol{x})[\delta(t)-\delta(t-\tau)] + \hat{y}\cdot\boldsymbol{j}_t\\
&\frac{1}{2\pi}(i\partial_t\partial_x\chi + v_c\partial_x\partial_x\chi) = \delta(\boldsymbol{x})[\delta(t)-\delta(t-\tau)] + \hat{y}\cdot\boldsymbol{j}_t.
\end{aligned}\tag{14}$$

Eq.(14) states that both tunneling and bulk transverse currents contribute to expand the charge and flux edge profiles. (The longitudinal bulk current causes internal compression and hence does not contribute to expand the edge profiles.)

By integrating out the bulk current fluctuations we obtain the following effective theory for the edge

$$\begin{aligned}
S &= S' + S_t\\
S' &= \int\frac{dkd\omega}{(2\pi)^2}\{\frac{(-ik\omega+v_ck^2)}{2\pi}|\phi_{k\omega}|^2 + \frac{(ik\omega+v_nk^2)}{4\pi}|\chi_{k\omega}|^2\\
&\quad + \frac{\gamma(k,\omega)}{4\pi}|\omega||k||2\phi_{k\omega}-\chi_{k\omega}|^2\}.
\end{aligned}\tag{15}$$

In Eq.(15)

$$\gamma(k_x,\omega) = \frac{1}{|k_x|}\int dk_y[\frac{k_x^2}{k_x^2+k_y^2}\sigma_{tt}(\boldsymbol{k},\omega)], \tag{16}$$

where $\sigma_{tt}(\boldsymbol{k},\omega)$ is the bulk transverse conductivity. The last term in Eq.(15) reflects the damping of the edge modes by bulk excitations. In the language of the two-mode model presented earlier ϕ and χ act as the charge and neutral mode respectively. In the clean limit $\sigma_{tt}(\boldsymbol{k},\omega) \sim 1/|\boldsymbol{k}|$ for $|\omega| << V_B|\boldsymbol{k}|$. In that case $\gamma(k,\omega) \sim 1/|k|$ and the last term in Eq.(15) changes the scaling property of the electron Greens function entirely. In the presence of disorder it is reasonable to assume that $\sigma_{tt}(\boldsymbol{k},\omega)$ saturates to a constant as $|\boldsymbol{k}|\to 0$. In that case $\gamma(k,\omega) \to$ *constant* at long wavelength and the last term of Eq.(15) is a marginal perturbation. For simplicity in the following we shall assume that $\gamma(k,\omega) = \gamma_0\theta(v_B|k|-|\omega|)$.

To compute the electron Greens function we calculate

$$G_e(\tau) = \frac{\int D[\phi]D[\chi]e^{-(S'+S_t)}}{\int D[\phi]D[\chi]e^{-S'}}. \tag{17}$$

In Eq. (17)

$$S_t = i\{[2\phi(0,0)-\chi(0,0)] - [2\phi(0,t)-\chi(0,t)]\}. \tag{18}$$

In the limit $\gamma_0 = 0$ Eq.(17) gives

$$G_e(t) \sim e^{i\frac{\pi}{2}\mathrm{sign}(t)}\frac{1}{t^2}. \tag{19}$$

The above result has the long time behaviors implied by Grayson *et al*'s experiment, and obeys $G_e(0^+) = -G_e(0^-)$ required by the electron Fermi statistics. For non-zero γ_0 the calculation is a little more involved. To proceed we define $\eta = 2\phi - \chi$ and $\zeta = 2\phi + \chi$. After integrating out ζ we obtain

$$G_e(\tau) = \frac{\int D[\eta]e^{-(S_\eta+S_t)}}{\int D[\eta]e^{-S_\eta}}, \tag{20}$$

where

$$\begin{aligned}
S_\eta &= \frac{1}{4\pi}\int\frac{dkd\omega}{(2\pi)^2}\{A(k,\omega)+\gamma(k,\omega)|\omega||k|\}|\eta_{k\omega}|^2\\
S_t &= i[\eta(0,0)-\eta(0,t)].
\end{aligned}\tag{21}$$

In Eq. (21) $A(k,\omega) = [k(-i\omega+v_ck)(i\omega+v_nk)]/(i\omega+v_ck+2v_nk)$. It is straightforward to show that

$$
\begin{aligned}
G_\eta(x,t) &= G_\eta^R + iG_\eta^I \\
G_\eta^R &= -4\pi \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \int_0^\infty \frac{d\omega}{2\pi} S(k,\omega) e^{-\omega|t|} \cos kx \\
G_\eta^I &= 4\pi \mathrm{sign}(t) \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \int_0^\infty \frac{d\omega}{2\pi} S(k,\omega) e^{-\omega|t|} \sin kx,
\end{aligned}
\tag{22}
$$

where

$$
S(k,\omega) = Im\{\frac{1}{A(k, i\omega + 0^+) + i\omega|k|\gamma(k,\omega)}\}. \tag{23}
$$

While G_η^R is responsible for the long-time decay of the electron Greens function, G_η^I fixes up the short time statistics. Unfortunately we will not be able to explicitly calculate the short time behavior of G_e. This is because such calculation requires knowledge of the bulk current-current correlation function at both low and high frequencies. To compute the long time behavior of G_η^R it is save to set both x and v_n to zero. We have evaluated G_η^R in Eq. (22) in the limit $\gamma_0 << 1$. The result reduces to Eq.(19) if $v_B < v_c$, and it gives $G_\eta^R(t) = constant - (2 + O(\gamma_0^2)) \ln|t|$ for $v_B > v_c$. Thus in the latter case the tunneling exponent receives an $O(\gamma_0^2)$ correction. Nonetheless for small γ_0 (or when $\sigma_{tt} << e^2/h$) such correction can be well within the experimental resolution.

In conventional wisdom as the filling factor approaches $\nu = 1/2$ through the sequence $\nu = \frac{n}{2n+1}$ the number of edge mode diverges. Therefore it is perhaps puzzling to see only two modes at $\nu = 1/2$. The two-mode model presented earlier precisely answers this question. Indeed, Eq. (15) can be viewed as the final product after infinite many neutral modes are integrated out in the $n \to \infty$ limit. (In general we expect a similar two-mode effective theory to be applicable to other compressible filling factors.)

An important omission in our discussion so far is the intrinsic and extrinsic dissipation (such as those from mode-mode interactions and phonons). Here we argue that dissipation can also prevent an edge mode from contributing to the long time decay of G_e. To make the point it is sufficient to consider the simplest case where a single edge mode (described by chiral field ϕ) is damped so that its chiral-boson action is given by

$$
S_c = \frac{1}{4\pi\nu} \int \frac{d\omega dk}{(2\pi)^2} k[-i\omega + vk - i\mathrm{sign}(k)\Gamma_k]|\phi_{k\omega}|^2. \tag{24}
$$

It is simple to show that the (real-time) Greens function is given by

$$
\begin{aligned}
G_\phi^R(t) &= 2\pi\nu \int_0^\Lambda \frac{dk}{2\pi k} e^{-(ivk+\Gamma_k)|t|} \cos kx \\
G_\phi^I(t) &= 2\pi\nu \mathrm{sign}(t) \int_0^\Lambda \frac{dk}{2\pi k} e^{-(ivk+\Gamma_k)|t|} \sin kx.
\end{aligned}
\tag{25}
$$

Eq. (25) suggests that while damping does not affect the short-time behavior of G_ϕ, it damps out the quantum fluctuation of ϕ at long time [13] and hence prevents it from contributing to the decay of G_e. Thus damping can also remove the contribution of neutral mode to α.

To summarize, we find that there is an energy scale, $v_n\Lambda_n$, above which $I \sim V^\alpha$ with $\alpha = 1/\nu$. As the temperature/voltage is lowered below $v_n\Lambda_n$ we expect to see a change in α caused by the neutral modes.

Finally we point out that under right condition the *double-layer* $\nu = 2$ state can exhibit $\alpha = 1/2$. However due to the spatial separation between the two edge modes and the fact that only the outer edge couples sufficiently to the tunneling electrode we do not expect the same for the single layer $\nu = 2$.

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